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A TENTATIVE PROGRAM OF JUNIOR HIGH-SCHOOL MATHEMATICS.*

By C. B. Walsh.

The statement is axiomatic and perhaps trite that the teaching of secondary-school mathematics is in a state of unrest. This condition has been aggravated, if not caused, by excessive criticism and this unstable state of affairs is reflected by the numerous new courses of study. It is consequently as inevitable as it is probably desirable that our curriculum in mathematics is to be reconstructed.

This reformation will be ephemeral and not worth while unless it addresses itself to the settlement of certain fundamental problems which doubtless are the worthy causes of this restlessness.

I refer to such searching queries as these:

(*a*) How shall we gain and maintain the interest of the so-called non-mathematical student?

(*b*) How shall we lessen the mortality in the mathematics courses of the ninth school year?

(*c*) How shall we enrich the practical content of our courses?

(*d*) How much mathematics shall be required of all pupils?

In part such questions must be answered by matters of method and we are not here concerned with method. Consideration of such inquiries, however, must be kept to the fore in moulding courses of study.

Problems of the type enumerated have, of course, always been of prime importance, but with the change in the character of the secondary schools they have become more insistent. Such an audience as this is so keenly aware of the nature of this change in high schools that I need lay no emphasis on the democratic trend of these institutions. Under our very eyes we have seen the aristocrat's academy expand into everybody's school with the concomitant change in function from preparation for college to preparation for life.

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* Read at the convention of the Association of Teachers of Mathematics in the Middle States and Maryland in Trenton, N. J., April 28, 1917.

The administrative adjustment to this situation has taken concrete form in the junior-senior high-school arrangement. Under this scheme the seventh, eighth and ninth school years (or junior high school) form a testing period by giving all students an extensive view of as many kinds of work as possible. Then, during the tenth, eleventh and twelfth school years (or senior high school) the survivors are offered in more intensive, elective courses, work suited to their individual needs and interests. This second stage might be termed prevocational. In a word we might refer to the junior high school as a period of opportunity and the senior high school as a period of preparation.

The motto of those of us who teach secondary school mathematics should be now, if ever, "Carpe diem." In this junior-senior high-school program we have an unparalleled opportunity to reorganize our curriculum completely and, I personally believe, in this new six-year course in mathematics a chance is given so to frame our course as to frustrate all legitimate criticism and to answer satisfactorily such problems as I presented at the outset. Obviously this cannot be done by spreading our four years' course over six, nor should it be done by bringing any of the grammar-school nor of the college mathematics as such into the secondary school course.

It does afford an opportunity to imitate the best that is in the foreign secondary-school curricula. It makes possible some such a program as Dr. D. E. Smith suggested in the MATHEMATICS TEACHER, Vol. IX., No. 2, for December, 1916. It allows us to furnish a period of testing and sampling in the junior high school, giving the student a casual acquaintance with the great phases of the subject, arithmetic, algebra and geometry, and to follow this by a period of preparation for higher work.

It is my privilege to outline for you in some detail a tentative program for junior high-school mathematics. Much of what I have to offer is a sort of report of an experiment of about four years' successful operation at the Ethical Culture School, New York City. Although we have not there a formal division into junior and senior high schools, our subject courses have long since followed the spirit of such a formation.

The portion of the mathematics course to which I wish especially to call your attention will be clear if you will allow me first to give in outline the full six-year program.

The work of the seventh, eighth and ninth school years in mathematics is required of all students and gives an extensive view of arithmetic, algebra and geometry, intuitive and demonstrational. The curricula of the tenth, eleventh and twelfth years are elective and furnish intensive courses in college preparatory mathematics, including solid geometry, trigonometry, and a notion of analytics or calculus or business arithmetic.

Our attention at present is to be focused on the first group of these courses and whatever of profit I may be able to present will come from considering the specific details of the work of the seventh, eighth, and particularly the ninth year.

For the seventh year there is little new for me to offer as the result of our experiment, but it seems to me that this course should present two phases of the subject: (*a*) Arithmetic and (*b*) Intuitive Geometry.

At this time what might be termed "grade-school arithmetic" should be completed. When the subject was continued through the eighth year many topics were included which would now be omitted, but that merely coincides with the consensus of opinions of the teachers of the subject independent of the time element. There still remains time for reasonable drill on the mechanical side of the subject, such as the fundamental operations with integers and fractions, probably emphasizing here the decimal fraction. Further time would be afforded for simple problem work in percentage and mensuration.

Connected with and growing out of this last topic is the so-called intuitive geometry. By means of simple constructions, drawing to scale, and the like, the student at this point should gain, through his mensuration work, a familiarity with the tools, *i. e.,* names and notions, which will greatly facilitate his later work in demonstrational geometry. Let him thus preface his work in the science of the subject by the art. History and pedagogy both point to that procedure.

In the eighth year there should be an extensive course in algebra. If you will permit me to indulge in a negative, I would add not the traditional symbol-juggling course. Remembering that this is a junior high-school required course including, therefore, future artists, carpenters, and others, with a sprinkling of college candidates, why, let me ask, should we drill these youngsters until they are professionally proficient in factoring most unusual expressions, removing ingenious nests of parentheses, simplifying a marvelous complexity of radicals?

No! Give them interesting, practical work in the subject! Let them catch the spirit of mathematics. Otherwise, wherein are we fulfilling the function of a junior high school?

Show the students that algebra is the shorthand of science. How? By the study of the formula. Show them that algebra is a tool for solving applied problems. How? By the study of the equation. Show them that mathematics has an elementary method within their grasp of representing statistics and arriving at approximate results. How? By the study of the graph. For all of this work a minimum amount of manipulation of the fundamental operations in algebra will be necessary as a means —not an end.

In our course at the Ethical School we introduce this work by a brief discussion of the historical extension of the number system until it includes negative numbers and letters. Then we apply these extensions to a study of the formula and the equation, of necessity including the work in fundamental operations in algebra, but introducing it only when needed, and using the graphic work throughout the course as one more means of expression and illustration. Through the proper selection of material for the formula work and problems resulting in the equations studied, mensuration is reviewed and extended, and much of the work formerly included in the eighth-grade arithmetic can be included from a different and, I submit, a saner point of view.

For brevity's sake, I refer you here to some notebooks (Graphisches Heft) here on exhibit, in which you will see some of the work of our pupils in connection with the study of graphs in this grade. It is our plan to have them record in this form simply one of a type of problem in graphs and such a notebook is to be kept throughout the high-school course. At the end, then, a pupil has a cumulative reference book of his graphic work. This plan, however, seems to have been frustrated by the fact that we are no longer able to import these notebooks, but surely others are available.

Again to see what power a pupil is expected to gain in algebra through such a course, I refer you to the MATHEMATICS TEACHER, Vol. VIII., No. 3, March, 1916, pp. 145, where you will find a set of problems grouped under the four headings—"Number Systems," "Equation," "Formula," and "Fundamental Operations"—selected from the materials we use in this course. We feel that pupils who can successfully handle such problems have gained a worth-while conception and mastery of algebra as shorthand method of generalization and a tool for the solution of applied problems, although such a pupil has not acquired the finesse in manipulation of unusual complexities of symbolism, not even to the degree demanded by college entrance requirements. That accomplishment we believe to be the task of the senior high school.

The traditional course in algebra, given in the ninth school year, first of the four-year high-school course, has not proved, to put it conservatively, a great success. I base that assertion on (1) the number of failures and (2) the lack of enthusiasm for the subject on the part of the majority of survivors. Our department gave itself over to the task some years ago of reconstructing this course to overcome those difficulties and to do so in the spirit of the junior high school, *i. e.*, to furnish the maximum variety of opportunity of testing abilities and interests.

We were further impressed with the fact that whatever practical or disciplinary value mathematics had to offer these young people lay not in the abstract symbol-juggling to which algebra had largely degenerated but rather in demonstrational geometry, if it could be adapted to the age of the pupils. Of course, we thought of correlated or fused mathematics, but I venture the assertion that not only could we not formulate such a course but although I have investigated most experiments in that direction I have yet to see a fusion that was not confusion, or a correlation that was not merely an incident of method or wholly artificial. Study of the subject has quite shattered my expectation of finding such a situation, because the nature of the topics makes it scientifically impossible. Those who argue that it is done in the Gymnasium or Lycée I refer to Professor Upton, of Teachers College, for his defense of the negative will be much more scholarly than mine.

Our solution of the problem was to give a course in demonstrational geometry with a deliberate introduction, a limited syllabus, and exercises of applied nature. I shall tax your patience to give some details of procedure under those three heads, (1) Introduction, (2) Syllabus, and (3) Exercises.

Let me preface those statements by the testimony that our experience has been that demonstrational geometry, inductively presented, made simple because scientific, is adapted to the age of pupils of the ninth school year.

The introduction to such a course should be very gradual. Familiarity with the tools of the subject might be increased by a discussion of the historic development of geometry from the art of the Egyptians to the science of the Greeks. Considerable construction work should be given. By means of numerous life-illustrations, measurements, experiments and the like the need and formation of a syllabus should be approached. Paper-folding experiments and measurements of angles by crude instruments should pave the way for the genuine scientific work. Then the first few propositions should be developed scientifically, but in a manner, though wholly convincing, yet lacking much of the rigor of form demanded later. Let matters of form grow with need. Therefore, we find it wise not to have pupils write any proofs until, let us say, four or five propositions have been developed. Then they will fall naturally into a desirable form. There is no dearth, but rather an embarrassing richness of exercises for them to write prior to this period of proof-writing. We have listed definitely some one hundred and fifty such problems.

A very important element of this course, which we call a "First Reading (or study) of Geometry," in the syllabus. It consists of only 64 propositions, covering in an extensive fashion the whole field of plane geometry. May I explain parenthetically here that those who go on with geometry beyond this course take what we term a "Second Reading" course, in which the syllabus is greatly augmented by the insertion of other theorems. Thus we preserve the original sequence, merely placing the added propositions between others in the First Reading.

We selected the limited syllabus on three bases: (1) Propositions needed for a logical sequence. Some of the work concerning regular polygons usually found in Book V. we had to retain because needed for later proof.

(2) Theorems interesting to young people were preserved for that reason. Such a reason alone would justify the maintenance of fact like the Pythagorean relation.

(3) Propositions rich in application were retained on that basis. The area propositions illustrate this point.

You may see at once that propositions on inequality would have no place in this First Reading. Such theorems as " Square of the bisection of an angle of a triangle equals . . ." would, of course, have to go.

Then the order of the syllabus differs considerably from the traditional. Our objects in this have been to obtain a sequence of topics more *natural* than the former division into five books permitted, to get an order more psychological in that the simpler work preceded the more difficult as far as possible, and finally more pedagogical in that the interest is enlisted by bringing work rich in applications forward as early in the course as possible. This means, for instance, that following congruence with its application to parallelism, comes areas, then similarity, and then all the study of the circle growing out of locus comes last.

Finally, the selection of exercises is an important matter in this ninth year. Here we have sought to make them so far as possible applications in a variety of fields, in contradistinction to those used in the Second Study of Geometry in the senior high school where the exercises are chiefly theoretic—of the college entrance type.

Throughout pupils make and use crude instruments by which they measure heights and distances. In such instruments the lack of mechanical refinements make more patent the geometrical principles and, moreover, the less able child may make them inexpensively. I refer to such instruments as simple theodolites, astrolabes, square or diagonal scales, quadrati baculi mensorum, and the like.

An excellent discussion of a few such instruments will be found in *School Science and Mathematics,* Vol. X. (1910), pp. 48 and 126, by William E. Stark, then head of our high school. Problems suitable for solution with such instruments might be found in *Teachers College Record,* March, 1909, and in many modern texts, such as Stone-Millis, Betz-Webb, Slaught-Lennes, etc.

If I have left the impression that because this is a geometry course and not a so-called fused course, algebra is omitted, the fault is mine and not one of the course. Algebra is introduced when needed, and that is often, for example: (1) The equation is used in connection with the sum of the angles of polygons, (2) fractions are introduced in connection with areas, (3) ratio and proportion in beginning similarity and (4) the elements of the theory of exponents are taken up in the applications of the work in similarity. Here logarithms are introduced. Of course, the treatment of this topic from a theoretic standpoint is meager, for we only consider enough of the theory to make their use an intelligent one, but by introducing logarithms here the extent of the applications is greatly increased and the pupils have the use of this important tool of calculation throughout the senior high school, or if they leave school they have a valuable asset therein. Of course, the slide rule is discussed and pupils are encouraged to procure and use one when they wish and can.

In connection with similar triangles, the three trigonometric ratios of sine, cosine and tangent are studied. This topic, too, greatly extends the field of applications.

If time permitted, I should like to dwell longer on this course and point out such matters as the rewording of many traditional theorems in which particular we have not infrequently adopted the phraseology of the children, and the method of presentation of the idea of similarity by which the application to shadows, map-making, etc., is made more apparent, and the presentation of locus from the algebraic point of view. Such discussion would, however, be taking an unwarranted license with the assigned subject.

By way of a concluding summary, let me repeat that the tentative curriculum in junior high-school mathematics which I proffer for your discussion is this:

Seventh year: Arithmetic and Intuitive Geometry.
Eighth year: Algebra.
Ninth year: Geometry.

This work I submit should be required of all students and be extensive rather than intensive, in order that, in the spirit of the junior high school, it should be practical and open to the students a variety of vistas, testing thus their ability and interest. To accomplish this I should stress in the algebra the equation, the graph and formula; in the geometry I should use a limited syllabus and enrich the course through applied exercises. The testimony I bring is that such a program decidedly decreases the mortality in the subject and greatly fires the enthusiasm.

In short, I finally submit that such a program, correctly administered, answers the questions of how much mathematics shall be required of all pupils, how shall we enrich the contents of our courses, how shall we save the non-mathematical, and, moreover, such a curriculum answers the appeal of the junior high school to give to all an opportunity of finding their interest and ability.

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